# Learning by Over-the-Air Training: Distributed Precoding for Cell-Free Massive MIMO

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Abstract—In this paper, we present a novel method of downlink precoding for cell-free massive multiple-input multiple-output (MIMO) systems using over-the-air (OTA) training. By drawing analogies between a cell-free massive MIMO system and an artificial neural network (ANN), we borrow the idea of backpropagation algorithm to optimize the precoders and combiners via OTA signal exchanges, without incurring channel state information (CSI) estimation or CSI aggregation over some backhaul lines. Numerical simulations show that our method outperforms the state-of-the-art methods in average sum-rate, is robust against pilot contamination, and has lower computational complexity.

Index Terms—Cell-free massive MIMO, distributed precoding, quasi-neural network, back-propagation algorithm

#### I. INTRODUCTION

Cell-free massive multiple-input multiple-output (MIMO) [1], [2], as a recently emerged physical layer technology that combines massive MIMO [3] and distributed access points (APs), can significantly outperform traditional cellular massive MIMO in some practical scenarios [4]. In a cell-free system, the APs jointly serve all user equipments (UEs), with each AP serving a cluster of UEs selected by some allocation scheme [5]. To fully leverage the capacity of cell-free massive MIMO, extensive researches have been devoted to cooperative precoding and combing [6], [7], pilot assignment [8], power allocation [4], and addressing practical hardware impairments [9], [10].

Cooperative precoding and combining design in a cell-free massive MIMO system can be achieved using a centralized approach or a distributed one. A centralized approach, e.g., the centralized zero-forcing (ZF) precoding [11] and minimum mean-square-error (MMSE) precoding [12], usually requires local channel state information (CSI) and the optimized precoder exchange via backhaul links between the central processing unit (CPU) and the APs. Owing to the high dimensionality of aggregated channels, however, the centralized methods involve computational complexity and the overhead of CSI exchange overwhelmingly high. A distributed approach only requires local CSI and limited or even no backhaul exchange for optimizing the precoders such as local ZF downlink precoding [11], SLNR precoding [13] and MMSE precoding [12]. But these algorithms underperform the centralized ones because only partial information can be obtained at each AP.

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The bi-directional training method uses over-the-air (OTA) signaling to exchanges the CSI implicitly and to optimize the precoders [6]. But it is sensitive to pilot contamination due to the estimation of the cross-term information that contains CSI.

This paper presents a distributed algorithm of cooperative precoding and combing for downlink transmission in cell-free massive MIMO systems. By introducing a power control factor, we convert a non-convex maximum weighted sum-rate (MWSR) problem with power constraint into an unconstrained optimization problem. We then model a cell-free massive MIMO system as a "quasi-neural network" [14] (Quasi-NN) by drawing analogies between the cell-free massive MIMO system and an artificial neural-network (ANN), based on which we propose the distributed quasi-network precoding (DQNP) algorithm. The DQNP algorithm requires no explicit channel estimation nor backhaul exchange and it can accommodate for various optimization objectives, including MWSR and MMSE. Simulation results verify the superior performance of the proposed algorithm over the state-of-art method [6].

# II. SYSTEM MODEL AND PROBLEM FORMULATION

# A. System Model

We consider a downlink cell-free massive MIMO network, where a set of APs  $\mathcal{L} = \{1, \dots, L\}$ , each equipped with  $M_t$ transmitting antennas, serve a set of UEs  $\mathcal{K} = \{1, \dots, K\}$ , each equipped with  $M_r$  receiving antennas. Assume that the network works in time-division-duplex (TDD) mode and the APs transmit the data streams  $s_k$  to the UE k. Based on some predefined pairing between the UEs and the APs, the CPU allocates some of the data streams  $\mathcal{S}_l \subseteq \mathcal{S} = \{s_1, \dots, s_K\}$ to AP l with  $N_l = |\mathcal{S}_l|$  denoting the number of its served UEs. AP l processes the signal  $\mathbf{s}_l \in \mathbb{C}^{N_l \times 1}$  generated from  $\mathcal{S}_l$ with precoding matrix  $\mathbf{P}_l = [\mathbf{p}_{l,1}, \dots, \mathbf{p}_{l,N_l}] \in \mathbb{C}^{M_t \times N_l}$  and transmits

$$\mathbf{x}_l = \mathbf{P}_l \mathbf{s}_l \in \mathbb{C}^{M_t \times 1}.$$
(1)

The received signal at UE k is

$$\mathbf{y}_{k} = \sum_{l=1}^{L} \mathbf{H}_{k,l} \mathbf{x}_{l} + \mathbf{n}_{k} \in \mathbb{C}^{M_{r} \times 1},$$
(2)

where  $\mathbf{n}_k \sim \mathcal{CN}(0, \sigma_k^2)$  is the additive white Gaussian noise (AWGN) and  $\mathbf{H}_{k,l} \in \mathbb{C}^{M_r \times M_t}$  denotes the downlink channel

between AP l and UE k. Subsequently, UE k uses combiner  $\mathbf{w}_k \in \mathbb{C}^{M_r \times 1}$  to recover  $s_k$  as:

$$\hat{s}_k = \mathbf{w}_k^H \mathbf{y}_k. \tag{3}$$

The output signal-to-interference-plus-noise ratio (SINR) for UE k is

$$\operatorname{SINR}_{k} = \frac{\left|\sum_{l \in \mathcal{L}_{k}} \mathbf{w}_{k}^{H} \mathbf{H}_{k,l} \mathbf{p}_{k,l}\right|^{2}}{\sum_{\bar{k} \in \mathcal{K} \setminus \{k\}} \left|\sum_{l \in \mathcal{L}_{\bar{k}}} \mathbf{w}_{k}^{H} \mathbf{H}_{k,l} \mathbf{p}_{\bar{k},l}\right|^{2} + \sigma_{k}^{2} \|\mathbf{w}_{k}\|^{2}},$$
(4)

where  $\mathcal{L}_k$  denotes the set of APs serving UE k. Hence the weighted sum-rate can be given by

$$R = \sum_{k \in \mathcal{K}} \omega_k \log_2 \left( 1 + \text{SINR}_k \right) \text{ bps/Hz}$$
 (5)

with  $\omega_k$ ,  $k = 1, \dots, K$  denoting the weight for UE k.

# B. Problem Formulation

This paper focuses on optimizing the precoding matrix  $\{\mathbf{P}_l\}_{l \in \mathcal{L}}$  and combiner  $\{\mathbf{w}_k\}_{k \in \mathcal{K}}$  to maximize the weighted sum-rate (5) subject to the unit power constraint for each AP, i.e.,

$$\max_{\substack{\{\mathbf{P}_l\}_{l\in\mathcal{L}},\{\mathbf{w}_k\}_{k\in\mathcal{K}}\\ \text{s.t.}}} \sum_{k\in\mathcal{K}} \omega_k \log_2\left(1 + \text{SINR}_k\right) \\ \|\mathbf{P}_l\|_F^2 \le 1, \quad (l\in\mathcal{L}),$$
(6)

where  $\|\cdot\|_F$  stands for the Frobenius-norm.

Since the precoder  $\mathbf{P}_l$  consists of both amplitude and direction of the precoding, it can be decomposed into two parts as

$$\mathbf{P}_{l} = e^{-|\boldsymbol{\theta}_{l}|} \frac{\mathbf{V}_{l}}{\|\mathbf{V}_{l}\|_{F}}.$$
(7)

Here  $\mathbf{V}_l$  is intended to control the direction of the transmitted signal and  $\theta_l$  is the power control factor for AP l with  $e^{-|\theta_l|} \in [0, 1]$  guaranteeing the power constraint.

Hence, the constrained optimization problem (6) can be reformulated as an unconstrained optimization problem:

$$\max_{\{\mathbf{V}_{l},\theta_{l}\}_{l\in\mathcal{L}},\{\mathbf{w}_{k}\}_{k\in\mathcal{K}}} \sum_{k\in\mathcal{K}} \omega_{k} \log_{2}\left(1+\mathrm{SINR}_{k}\right), \qquad (8)$$

which can be solved by a distributed algorithm. The key is to the so-called quasi-NN, which draws analogies between a cell-free network and an ANN as explained in the next.

# C. The Analogies Between a Cell-Free Network and an ANN

The topology of a cell-free massive MIMO system is shown in Fig. 1, where the links are differently colored to indicate that a signal dedicated to some UE may be allocated to multiple APs. We follow the idea proposed for relay network communications in [14] and observe that it is similar to an ANN [15]: *i*). The antennas of each AP and UE are analogous to the neurons in the hidden layers of an ANN. The beamforming weights  $\mathbf{P}_l$ , channel weights  $\mathbf{H}_{k,l}$ , and the combining weights  $\mathbf{w}_k$  are similar to the connection weights for a four-layer ANN. *ii*). The streams for each AP and the combined stream can be regarded as the input layer and output



Fig. 1. The Analogies Between a Cell-Free Network and an ANN

layer. *iii*). The downlink transmission of pilots is analogous to the propagation of training samples in an ANN.

However, several practical limitations in a cell-free massive MIMO system make it different from an ANN: *i*). Pilot transmission in a cell-free massive MIMO system is contaminated by AWGN, while data processing in an ANN is typically error-free. *ii*). In a cell-free massive MIMO system, the channel weights are unknown and deterministic, whereas all connection weights and biases in an ANN are adjustable. *iii*). Data and weights in a cell-free massive MIMO system are complex values, but an ANN is typically real-valued.

Hence we follow the term used in [14], [15], in which the idea of Quasi-NN was orginally proposed to describe a relay network, and refer to a cell-free massive MIMO system as a Quasi-NN due to its similarities and differences to an ANN. Inspired by the backpropagation (BP) algorithm, we propose the distributed quasi-neural network precoding (DQNP) algorithm to optimize (8) distributedly.

# III. THE DISTRIBUTED QUASI-NEURAL NETWORK Algorithm

In this section, we present the DQNP algorithm with MWSR criterion via pilot training to optimize problem (8). Maximizing the output SNR amounts to minimizing the system MSE [16, Equation (13)] that

$$SNR = \frac{1}{MSE} - 1.$$
(9)

This holds for the well-known MMSE receiver, which can be implemented with downlink pilot sequences as:

$$\mathbf{w}_k = \left(\mathbf{Y}_k \mathbf{Y}_k^H\right)^{-1} \mathbf{Y}_k \mathbf{s}_k^*.$$
(10)

Here,  $\mathbf{s}_k \in \mathbb{C}^{\tau \times 1}$  and  $\mathbf{Y}_k \in \mathbb{C}^{M_r \times \tau}$  denote the downlink  $\tau$ -length pilot sequence and the received signal by UE k, respectively.

According to (9) can be approximated as the weighted sumrate in (8):

$$R = \sum_{k=1}^{K} \omega_k \log\left(\frac{1}{\frac{1}{\tau} \sum_{i=1}^{\tau} |s_k(i) - \hat{s}_k(i)|^2}\right), \quad (11)$$

where we use the approximation that  $MSE = \frac{1}{\tau} \sum_{i=1}^{\tau} |s_k(i) - \hat{s}_k(i)|^2$ , which is indeed asymptotically accurate as  $\tau \to \infty$ .

Hence problem (8) can be reformulated as:

$$\min_{\{\mathbf{V}_{l},\theta_{l}\}_{l\in\mathcal{L}}} J \triangleq \sum_{k=1}^{K} \omega_{k} \log \left(\frac{1}{\tau} \sum_{i=1}^{\tau} |s_{k}(i) - \hat{s}_{k}(i)|^{2}\right).$$
(12)

As J is a function of  $\mathbf{V}_l$ 's and  $\theta_l$ 's, we present in below the derivative  $\frac{\partial J}{\partial \mathbf{V}_l^*}$  and  $\frac{\partial J}{\partial \theta_l}$  with the time index t omitted for notational simplicity.

**Proposition 1.** The derivative respect to the direction precoder  $V_l$  of AP l is,

$$\frac{\partial J}{\partial \mathbf{v}_{ln}^H} = \frac{\partial J}{\partial x_{ln}^*} \frac{\partial x_{ln}^*}{\partial \mathbf{v}_{ln}^H} + \frac{\partial J}{\partial x_{ln}} \frac{\partial x_{ln}}{\partial \mathbf{v}_{ln}^T},$$
(13)

where  $\mathbf{v}_{ln}$  is the *n*-th column of  $\mathbf{V}_l^T$  such that  $\mathbf{V}_l = [\mathbf{v}_{l1}^T; \cdots; \mathbf{v}_{lM_t}^T]^T$  and  $x_{ln}$  is the *n*-th element of transmitted signal  $\mathbf{x}_l$  given in Eq. (1). Meanwhile,

$$\frac{\partial J}{\partial \mathbf{x}_l^*} = \sum_{k=1}^K \mathbf{H}_{k,l}^H \mathbf{w}_k \frac{\partial J}{\partial \hat{s}_k^*},\tag{14}$$

where

$$\frac{\partial J}{\partial \hat{s}_k^*} = \frac{\omega_k \left( s_k - \hat{s}_k \right)}{\sum_{i=1}^{\tau} |s_k(i) - \hat{s}_k(i)|^2}.$$
 (15)

And in (13)

$$\frac{\partial x_{ln}}{\partial \mathbf{v}_{ln}^H} = e^{-|\theta_l|} \frac{-\mathbf{v}_{ln}^T \mathbf{s}_l \operatorname{tr} \left(\mathbf{V}_l \mathbf{V}_l^H\right)^{-\frac{1}{2}} \mathbf{v}_{ln}^T}{2 \|\mathbf{V}_l\|_F^2}, \qquad (16)$$

$$\frac{\partial x_{ln}^*}{\partial \mathbf{v}_{ln}^H} = e^{-|\theta_l|} \frac{2\mathbf{s}_l^H \|\mathbf{V}_l\|_F - \mathbf{v}_{ln}^H \mathbf{s}_l^* \operatorname{tr} \left(\mathbf{V}_l \mathbf{V}_l^H\right)^{-\frac{1}{2}} \mathbf{v}_{ln}^T}{2 \|\mathbf{V}_l\|_F^2}.$$
 (17)

The derivative

$$\frac{\partial J}{\partial \theta_l} = -2\Re \left\{ \operatorname{sign}(\theta_l) e^{-|\theta_l|} \left( \frac{\partial J}{\partial x_l^*} \right)^H \frac{\mathbf{V}_l}{\|\mathbf{V}_l\|_F} \mathbf{s}_l \right\}, \quad (18)$$

with  $sign(\cdot)$  denoting the sign function that outputs 1 if the input is positive and outputs -1 if the input is negetive.

Proof. Represent eq. (12) as

$$J = \sum_{k=1}^{K} \omega_k \log\left(\frac{1}{\tau} \sum_{i=1}^{\tau} \left(s_k(i) - \hat{s}_k(i)\right) \left(s_k^*(i) - \hat{s}_k^*(i)\right)\right).$$
(19)

Differentiating it with respect to  $\hat{s}^*(i)$ ,  $i = 1, \dots, \tau$ , we obtain (15).

By (2) and the definition of derivative on complex variables, we have the following derivatives

$$\frac{\partial \mathbf{y}_k^H}{\partial \mathbf{x}_l^*} = \mathbf{H}_{k,l}^H, \quad \frac{\partial \mathbf{y}_k^T}{\partial \mathbf{x}_l^*} = 0.$$
(20)

By the chain rule, we can prove (14)

$$\frac{\partial J}{\partial \mathbf{x}_l^*} = \sum_{k=1}^K \left( \frac{\partial \mathbf{y}_k^H}{\partial \mathbf{x}_l^*} \cdot \frac{\partial J}{\partial \mathbf{y}_k^*} + \frac{\partial \mathbf{y}_k^T}{\partial \mathbf{x}_l^*} \cdot \frac{\partial J}{\partial \mathbf{y}_k} \right) = \sum_{k=1}^K \mathbf{H}_{k,l}^H \mathbf{w}_k \frac{\partial J}{\partial \hat{\mathbf{s}}_k^*}.$$
(21)
Since  $x_{ln} = e^{-|\theta_l|} \mathbf{v}_{ln}^T \mathbf{s}_l, n = 1, \cdots, M_t$  and  $\|\mathbf{V}_l\|_F^2 = \mathbf{v}_{ln}^T \mathbf{v}_l \mathbf{v}_l^T \mathbf{s}_l$ 

tr  $(\mathbf{V}_l \mathbf{V}_l^H)$ , we can prove derivatives (16) and (17) by the quotient rule.

By (1) and (7), we have

$$\frac{\partial \mathbf{x}_l}{\partial \theta_l} = -\operatorname{sign}(\theta_l) e^{-|\theta_l|} \frac{\mathbf{V}_l}{\|\mathbf{V}_l\|_F} \mathbf{s}_l.$$
 (22)

By the chain rule and the conjungate property, we can prove (18).

Proposition 1 provides insights on designing a distributed precoding algorithm:

1) Each UE locally updates the local MMSE combiner by eq. (10).

2) Each UE transimits the derivative  $\frac{\partial J}{\partial \hat{s}_k^*}$  as given in (15) using  $\mathbf{w}_k$  for uplink precoding:

$$\mathbf{x}_{k}^{\mathrm{ul}} = \mathbf{w}_{k} \frac{\partial J}{\partial \hat{s}_{k}^{*}}.$$
(23)

And AP *l* receives

$$\mathbf{y}_{l}^{\mathrm{ul}} = \sum_{k=1}^{K} \mathbf{H}_{k,l}^{H} \mathbf{w}_{k} \frac{\partial J}{\partial \hat{s}_{k}^{*}} + \mathbf{n}_{l}^{\mathrm{ul}}, \qquad (24)$$

where  $\mathbf{n}_l^{\mathrm{ul}}$  is the AWGN at AP l with elements distributed as  $\mathcal{CN}(0, \sigma_l^2)$ . Since the first term is the derivative  $\frac{\partial J}{\partial \mathbf{x}_l^*}$  as given in eq. (14), AP l can obtain an approximation of derivative  $\frac{\partial J}{\partial \mathbf{x}_l^*}$  by an uplink transmission without knowing CSI. Although  $\mathbf{y}_l^{\mathrm{ul}}$  is contaminated by the AWGN, it hardly affects the DQNP algorithm owing to the average operation with a length- $\tau$  pilot sequence when in the practical systems.

3) Since  $\mathbf{s}_l$ ,  $\mathbf{V}_l$ , and  $\theta_l$  are locally available, AP *l* can obtain derivatives  $\frac{\partial J}{\partial \mathbf{V}_l^*}$  and  $\frac{\partial J}{\partial \theta_l}$  by eq.(14) ~ (18) without CSI.

The results given in Proposition 1 are based on one pilot sample. For a pilot sequence with length  $\tau$ , we can average the  $\tau$  derivatives on each sample to reduce the effect of transmission noise. We have

$$\nabla \bar{\mathbf{V}}_{l} = \frac{1}{\tau} \sum_{i=1}^{\tau} \frac{\partial J(i)}{\partial \mathbf{V}_{l}(i)},$$
(25)

$$\nabla \bar{\theta}_l = \frac{1}{\tau} \sum_{i=1}^{\tau} \frac{\partial J(i)}{\partial \theta_l(i)}.$$
(26)



Fig. 2. Illustration of the Distributed Quasi-neural Network Over-the-air Iteration

We can apply the momentum gradient descent method with multiple sets of pilot sequences:

$$\nabla \mathbf{V}_l(t) = \beta \nabla \mathbf{V}_l(t-1) + (1-\beta) \nabla \bar{\mathbf{V}}_l(t), \qquad (27)$$

$$\nabla \theta_l(t) = \beta \nabla \theta_l(t-1) + (1-\beta) \nabla \bar{\theta}_l(t).$$
 (28)

Here,  $\beta \in (0, 1)$  is the hyper-parameter for derivative updating and  $t \in \{1, \dots, T\}$  is the pilot sequence index. The weights for precoders are updated by

$$\mathbf{V}_{l}(t) = \mathbf{V}_{l}(t-1) - \alpha \nabla \mathbf{V}_{l}(t), \qquad (29)$$

$$\theta_l(t) = \theta_l(t-1) - \alpha \nabla \theta_l(t).$$
(30)

where  $\alpha$  is the step size, which should be chosen to strike a balance between stability and speed of convergence (see [17] for more details).

We summarize the DQNP algorithm in Algorithm 1, with implementation illustrated in Fig. 2. It requires a downlink signaling resource and an uplink signaling resource for a training minislot. In each training minislot, the cell-free massive MIMO system propogates the training pilots forward in the downlink and the derivatives backward in the uplink. And each AP updates local weights  $V_l$  and  $\theta_l$  according to the derivatives without knowing the CSI. Multiple training minislots are needed to achieve a desired performance before data transmission.

From the computational complexity perspective, the DQNP algorithm only requires matrix multiplication with linear complexity  $\mathcal{O}(M_t)$ , while local ZF [11], local MMSE [12], and Distributed-OTA [6] requires matrix inverse with complexity of  $\mathcal{O}(M_t^3)$ .

**Remark 1.** The DQNP algorithm can be easily extended to a variety of objective functions, such as MMSE. And the combiners can also be optimized via the DQNP algorithm with the derivative of combiner  $\mathbf{w}_k$ :

$$\frac{\partial J}{\partial \mathbf{w}_k^*} = \mathbf{y}_k \left( s_k - \hat{s}_k \right)^*.$$
(31)

The average operation with length- $\tau$  pilot is

$$\nabla \bar{\mathbf{w}}_k = \frac{1}{\tau} \sum_{i=1}^{\tau} \frac{\partial J(i)}{\partial \mathbf{w}_k(i)}.$$
(32)

# Algorithm 1 DQNP Algorithm (Weighted Sum-Rate)

**Input:** Pilot Sequence  $\mathbf{s}(t), t \in \{1, \dots, T\}$ 

**Output:** Precoder  $\mathbf{V}_l, \theta_l, l \in \mathcal{L}$ 

**Initialize**:  $\mathbf{V}_l(0)$ , and  $\theta_l(0)$ , set t = 0;

- For t < T do:
  - 1.  $t \leftarrow t + 1;$
  - 2. **DL**: Each AP transmits beamformed signal  $\mathbf{x}_l$  by (1).
  - 3. Each UE computes MMSE receiver  $\mathbf{w}_k$  as (10).
  - 4. Each UE recovers  $\hat{s}_k$  and calculates loss J with (3) and (12).
  - 5. UL: Each UE transmits uplink signal  $\mathbf{x}_k^{\text{ul}}$  in (23) and (15).
  - 6. Each AP computes derivative  $\frac{\partial J}{\partial \mathbf{V}_l}$  and  $\frac{\partial J}{\partial \theta_l}$  by Proposition 1.
  - 7. Each AP updates  $V_l$  and  $\theta_l$  by (29) and (30).

End For

And the combiners are updated with the momentum gradient descent as:

$$\nabla \mathbf{w}_k(t) = \beta \nabla \mathbf{w}_k(t-1) + (1-\beta) \nabla \bar{\mathbf{w}}_k(t), \quad (33)$$

$$\mathbf{w}_k(t) = \mathbf{w}_k(t-1) - \alpha \nabla \mathbf{w}_k(t).$$
(34)

In summary, the DQNP algorithm is executed over-the-air, using training pilot sequences to implement BP algorithm by exploiting the similarity between a cell-free massive MIMO network and an ANN. And it requires no CSI. Moreover, the DQNP algorithm is robust against the pilot contamination problem as verified by the simulation result in the next section.

## **IV. SIMULATION RESULTS**

Using a simulation setting similar to [6], we simulate a network consisting of L = 100 four-antenna APs located in a square grid with inter-site distance 100 meters and height 10 meters. The APs serve K = 50 two-antenna UEs randomly located in the area. We consider the Rayleigh fading channel model:  $\operatorname{vec}(\mathbf{H}_{k,l}) \sim \mathcal{CN}(0, \delta_{k,l}\mathbf{I}_{M_rM_t})$ , where  $\delta_{k,l}[\mathrm{dB}] = -30.5 - 36.7 \log_{10}(r_{k,l})$  and  $r_{k,l}$  is the distance between AP l and UE k in meter. The AP's transmit power is set to be 30 dBm; the UEs' transmit power is 20 dBm; the noise power  $\{\sigma_l^2 = -95 \text{ dBm}\}_{l \in \mathcal{L}}$  and  $\{\sigma_k^2 = -95 \text{ dBm}\}_{k \in \mathcal{K}}$ . We use the sum rate, i.e.,  $\omega_k = 1$ ,  $k \in \mathcal{K}$  in (5), to evaluate the performance of the system based on the average of 100 Monte Carlo simulations with different channel realizations and random drops of the UE locations.

In the first simulation, 50 orthogonal 128-length pilots are adopted for the 50 UEs. For the proposed DQNP algorithm, two cases are simulated: i) each UE is served by all the 100 APs, and ii) each UE is served by only the 20 nearest APs. Their sum rate performances are shown by the circled solid lines and circled dash lines in Fig. 3, respectively, from which we see that proper UE-AP pairing can expedite the convergence. In [6], the state-of-the-art Distributed-OTA method only considered the first case. Fig. 3 shows that the DQNP algorithm can converge to a higher sum-rate than the



Fig. 3. Average Sum-rate versus Pilot Training Iteration

state-of-art method [6]. Although the DQNP algorithm converges slower, it is worth noting that it requires one downlink transmission and one uplink transmission per iteration, while the Distributed-OTA [6] requires two uplink and one downlink transmission per iteration.



Fig. 4. Average Sum-rate versus Pilot Length  $\tau$ 

The second example simulates both non-orthogonal random pilots and orthogonal pilots with varying length  $\tau$  as indicated by the circle-marked and diamond-marked lines respectively in Fig. 4. For  $\tau = 32$  ( $\tau < K = 50$ ), using orthogonal pilots will suffer from pilot contamination especially hard since identical pilots will unavoidably be assigned to more than one UEs, which explains why both the DQNP and the distributed-OTA (the solid lines with diamond markers) see severe sum-rate decrease as  $\tau$  reduces from 64 to 32. Compared with the distributed-OTA [6] the DQNP method with random pilots has performance closer to that with orthogonal pilots for  $\tau \geq 64$  (compare the circled/diamond solid lines in blue versus the ones in orange), which indicates that the DQNP method is more robust to pilot contamination caused by the nonorthogonal pilots. It is also seen that proper UE-AP pairing can reduce the impact of pilot contamination.

### V. CONCLUSION

In this paper, we draw analogies between a cell-free massive MIMO system and an ANN and borrow the idea of the backpropagation algorithm to maximize the weighted sum-rate of a cell-free massive MIMO system via OTA training. The resultant algorithm, i.e, the so-called DQNP algorithm, can distributively optimize the precoders and combiners by propagating training pilots forward in the downlink and feeding derivatives backward in the uplink during the iterative OTA training, which involves no explicit estimation or aggregation of the channel information but has superior performance.

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